Classification of distributive/thin representations via incidence algebras

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Overview

1. Modules with some finiteness properties and the distributive property coincide over finite dimensional algebras.
2. Incidence algebras produce examples of modules with those finiteness properties.
3. Deformation of incidence algebras, incidence algebras and cohomology.
4. Deformations of incidence algebras "=" incidence like algebras.
5. Application I: Characterization of incidence algebras via distributive/thin modules.
6. Application II: Classifications of thin modules.

Modules with finitely many orbits

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Theorem

Let M be an R-module. We consider the following conditions.

1. M has finitely many orbits.
2. M has finitely many submodules.
3. M has finite length and has not subfactor isomorphic to $T \oplus T$, where T is simple R-module with infinite endomorphism ring $\mathrm{End}_R(T)$.

Then we have the implications: (1) $\Rightarrow$ (2) $\Leftrightarrow$ (3). Moreover, if R is semilocal, then (3) $\Rightarrow$ (1).

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A left R-module M is distributive if for any submodules A, B and C of M, $A \cap (B + C) = A \cap B + A \cap C$.

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Corollary

Let A be a finite dimensional k-algebra, where k is an infinite field. Then an A-module M has finitely many orbits if and only if it has finitely many submodules, if and only if it is left artinian and distributive.

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The corollary follows since in this case A is semilocal and for every

Incidence Algebras

Definition: The **incidence algebra** $I(P, R)$ of the locally finite partially ordered set P over the commutative ring with identity R is

$$I(P, R) = \{f : P \times P \to R | f(x, y) = 0, \text{ if } x \not\leq y\},$$

with the operations given by

$$\begin{aligned}(f + g)(x, y) &= f(x, y) + g(x, y), \\ (f \cdot g)(x, y) &= \sum_{x \leq z \leq y} f(x, z) \cdot g(z, y) \quad \text{and} \\ (r \cdot f)(x, y) &= r \cdot f(x, y),\end{aligned}$$

for $f, g \in I(P, R)$ with $r \in R$ and $x, y, z \in P$.

Alternate definition

$I(P, R)$ is generated by the functions

$$f_{xy}(u, v) = \begin{cases} 1 & \text{if } u = x, v = y \\ 0 & \text{otherwise} \end{cases}$$

and

$$f_{xy} \cdot f_{zt} = \delta_{yz} f_{xt}$$

Incidence algebras as strutural matrix algebras

We can view the incidence algebra $I(P, k)$ as a subalgebra of the matrix algebra $M_{|P|}(k)$ by considering the set of matrices having arbitrary entries at a set of prescribed positions (i, j) and 0 elsewhere. In this case, we call the incidence algebra a **structural matrix algebra**.

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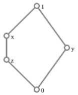
For example, given the poset



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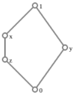

its corresponding structural matrix algebra is

$$\begin{pmatrix} R & R & R & R & R \\ 0 & R & 0 & 0 & R \\ 0 & 0 & R & R & R \\ 0 & 0 & 0 & R & R \\ 0 & 0 & 0 & 0 & R \end{pmatrix}$$

The defining representation for $I(P, k)$

Given an incidence algebra $I(P, k)$ viewed as a structural matrix algebra, we consider the action of $I(P, k)$ on the column vectors k^n. This gives a representation of the incidence algebra $I(P, K)$ which we call **the defining representation** of $I(P, k)$.

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The defining representation of $I(P,k)$ is distributive and thin.

Examples of Modules with finitely many orbits

Let $I(P,k)$ be an incidence algebra with k an infinite field. It is known that the projective indecomposable of $I(P,k)$ are given by $P_y = \mathrm{span}_k\{f_{xy} | x \leq y\}$.

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The proposition follows from the fact that since $\mathrm{Hom}_k(P_x, P_y) = f_{xx} I(P,k) f_{yy} = k f_{xy}$ is one dimensional, the multiplicity $[P_y : S_x] \leq 1$, and so the square-free condition of distributivity is satisfied.

Combinatorial algebras

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Corollary

Let A be a basic k-algebra(where k is infinite). If A is semidistribituve, then A has finitely many ideals. Conversely, if A is is acyclic, then projective indecomposable A-modules are distributive, equivalently, they have finitely many submodules, equivalently, they have finitely many orbits.

Geometric realization of a poset P

To every poset P, we associate a simplicial complex $\Delta(P)$ called **the geometric realization** of P as follows. The vertices of $\Delta(P)$ are the elements of P and the faces of $\Delta(P)$ are the chains of P.

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Let

$$\mathbb{Z}C_n = \mathbb{Z}^{\{(s_0,\cdots,s_n):s_0<\cdots<s_n, s_i\in P\}}.$$

Then we get a chain complex

$$\partial_n : \mathbb{Z}C_n \to \mathbb{Z}C_{n-1}$$

realizing singular homology with boundary map given by

$$\partial_n(s_0, s_1, s_2, \cdots, s_n) = \sum_{i=0}^{n} (-1)^i (s_0, s_1, \cdots, s_{i-1}, s_{i+1}, \cdots, s_n).$$

Cohomology

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$$(\partial_n)^* : E^{n-1} \to E^n$$

with $E^n = Hom_{\mathbb{Z}}(\mathbb{Z}C_n, k^*)$ and boundary map

$$(\partial_n)^*(f)(s_0, \cdots, s_n) = \prod_{i=0}^{n} f(s_0, \cdots, s_{i-1}, s_{i+1}, \cdots, s_n)^{(-1)^i}.$$

This yields cohomology groups

$$\boldsymbol{H^i(P, k^*)}.$$

An element $f \in I(P, k)$ is multiplicative if (i) $f(x, y) \neq 0$ for all $x \leq y \in P$ and (ii) $f(x, y)f(y, z) = f(x, z)$ for all $x \leq y \leq z \in P$

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Theorem (Feinberg)

If P is lower finite, then the isomorphism classes of faithful distributive $I(P,k)$-modules are in one to one correspondence with the elements of $H(P,k) = Z(P,k)/B(P,k)$, where
$\boldsymbol{Z(P,k)} = \{\text{The set of all multiplicative functions of } I(P,k)\}$
and
$\boldsymbol{B(P,k)} = \{f \in Z(P,k) | \exists \alpha : P \to k \setminus \{0\}, f(x,y) = \alpha(x)\alpha(y)^{-1}\}$.

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Theorem

If P is a finite poset, then the isomorphism classes of faithful distributive $I(P,k)$-modules are the elements of $H^1(P,k^*)$

Deformations of Incidence Algebras

Let $\lambda : \{(x, y, z)|x, y, z \in P, x \leq y \leq z\} \rightarrow k^*$ and denote $\lambda(x, yz)$ by λ^y_{xz}.

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Let $\lambda : \{(x, y, z) | x, y, z \in P, x \leq y \leq z\} \to k^*$ and denote $\lambda(x, yz)$ by λ_{xz}^{y}.

The algebra $I_\lambda(P, k)$ is defined as $I_\lambda(P, k) = I(P, k)$ as a vector space, but with multiplication $*_\lambda$ on $I(P, k)$ given by

$$f_{xy} *_\lambda f_{yz} = \lambda_{xz}^{y} \cdot f_{xz}$$

and

$$f_{xy} *_\lambda f_{tz} = 0$$

when $y \neq t$. We call $I_\lambda(P, k)$ a deformation of the incidence algebra $I(P, k)$.

Classification of deformations of Incidence Algebras

Proposition

$I_\lambda(P,k)$ is an associative algebra if and only if

$$\lambda_{xz}^{y} \cdot \lambda_{xt}^{z} = \lambda_{xt}^{y} \cdot \lambda_{yt}^{z}$$

for any $x \leq y \leq z \leq t$, that is if and only if λ is a 2-cocycle(i.e., $\lambda \in ker(\delta^3)$.
Furthermore, if $\lambda^{-1}\mu \in Im(\delta^2)$(i.e., λ and μ differ by a coboundary) then $I_\lambda(P,k) \cong I_\mu(P,k)$.

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Theorem

The isomorphism classes of deformations $I_\lambda(P,k)$ of incidence algebras of the poset $(P,\leq)$ are in one-to-one correspondence with the space $H^2(\Delta(P),k^*)/Aut(P,\leq)$ of orbits of the action of $Aut(P,\leq)$ on $H^2(\Delta(P),k^*)$.

Characterization of deformations of incidence algebras

Theorem

If $I_\lambda(P, k)$ has a faithful distributive representation M, then $I_\lambda(P, k) \cong I(P, k)$ is a trivial deformation. Moreover, the faithful distributive representations of $I(P, k)$ are in 1-1 correspondence with $H^1(P, k^*)$; more, precisely, $M = M_\alpha$ is a representation exactly when the α is 1-cocylce, and M_α is uniquely determined up to isomorphism by the cohomology class $[\alpha] \in H^1(P, k^*)$. Under this bijection, the defining representation of $I(P, k)$ corresponds to the trivial 1-cocycle $[\alpha] = [1]$.

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This follows from the fact that if M is a faithful distributive representation of $I_\lambda(P,k)$, then $f_{yy}M = km_y$ for some $m_y \in M$ and $f_{xy}km_y \subseteq km_x$. So, we can define $\alpha \in E^1$ by $f_{xy} \cdot m_y = \alpha_{xy} m_x$ for $x \leq y$. Thus, $(\alpha_{xy})_{x,y}$ determine the representation M. Hence the representation condition $(f_{xy} *_\lambda f_{yz}) \cdot m_z = f_{xy} \cdot (f_{yz} \cdot m_z)$ yields $\lambda^y_{xz}\alpha_{xz} = \alpha_{xy}\alpha_{yz} \Rightarrow \lambda = \delta^2(\alpha)$, and so $[\lambda] = [1]$ in $H^2(P,k^*)$.

Characterization of deformations of incidence algebras(Cont'd)

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Theorem

Let A be a finite dimensional basic k-algebra with k infinite. Then the following are equivalent.

1. A is a deformation of an incidence algebra.
2. A is locally hereditary and semidistributive.
3. A has finitely many two sided ideals and is locally hereditary.

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Let A be a basic algebra. The following are equivalent

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Corollary

Let A be a finite dimensional algebra. Then A is a structural matrix algebra or equivalently, an incidence algebra of a quasi-ordered set if and only if either conditions (2) or (3) of the above theorem holds.

“Generic” classification of distributive/thin representations

Theorem

Let A be a finite dimensional basic algebra. Let M be a representation of A. Assume either A is acyclic and M is distributive, or M is thin. Then $A/\mathrm{ann}(M)$ is an incidence algebra $I(P,k)$ of some poset P. Furthermore, M can be realized as the defining representation of $A/\mathrm{ann}(M)$.

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A change of an incidence algebra basis in $A/\mathrm{ann}(M)$ realizes M as the defining representation of $I(P,k)$ as follows. Take a basis $(f_{xy})_{x\leq y}$ of $A/\mathrm{ann}(M)$ and let M be given by the 1-cocyle α(that is α is mutiplicative: $\alpha_{xz} = \alpha_{xy}\alpha_{yz}$, $x \leq y$). Let $(m_x)_{x\in P}$ be a basis of M with $f_{xy}\cdot m_y = \alpha_{xy}m_x$. By defining $e_{xy} = \frac{1}{\alpha_{xy}}f_{xy}$, we see that $e_{xy}\cdot e_{yz} = e_{xz}$ and $e_{xy}\cdot m_y = m_x$. Thus, this change of basis of $A/\mathrm{ann}(M)$ realizes M as the defining representation of $I(P,k)$.

Consequence of the "generic" classification

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So, let M be any thin representation over any algebra A or any distributive representation over any acyclic algebra A. Then M is parametrized by the pair (I, α) where $I = \mathrm{ann}(M)$ is an ideal of A for which A/I is isomorphic to an incidence algebra of a poset $P = P(A/I)$ and α is an element of $H^1(P(A/I), k^*)$.

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The distributive representations, equivalently, the thin representations of an incidence algebra $I(P, k)$ are in 1-1 correspondence with the set $\bigsqcup_{(S, \leq_S) \text{ closed}} H^1(\Delta(S, \leq_S), k^*)$.

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Corollary

Let A be an acyclic algebra which is basic. Then any distributive representation M is completely determined by $I = \mathrm{ann}(M)$ and an element $\alpha \in H^1(\Delta(P_{A/I}), k^*)$, where $P_{A/I}$ is the poset associated canonically with the acyclic quiver of the algebra A/I.

This work is in collaboration with

Miodrag C. Iovanov at the The University of Iowa

The paper can be found in the math arxiv at

https://arxiv.org/abs/1702.03356

THANK YOU!!!